ME 301 THEORY OF MACHINES I
SOLVED PROBLEM SET 1

PROBLEM 1.

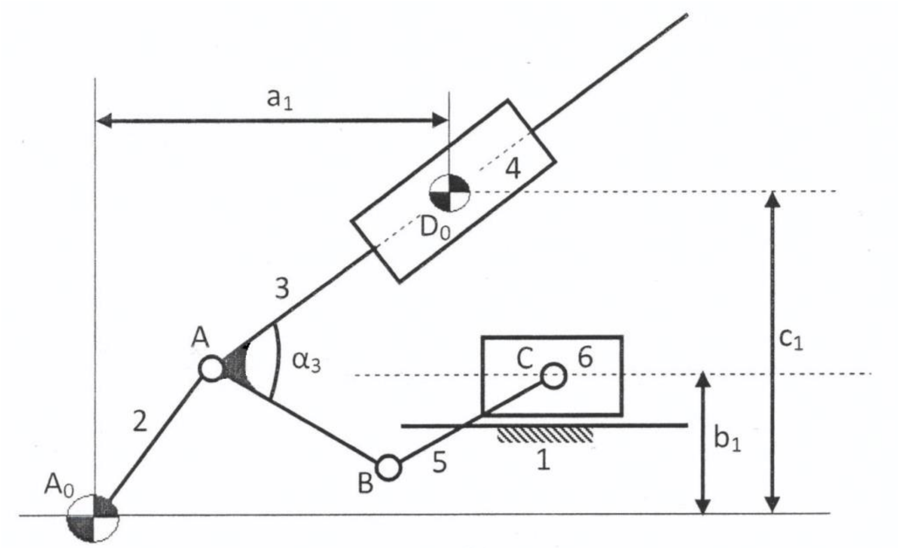
For the planar mechanism given below:

a) Find the degree of freedom, the number of independent loops, and the total number of required joint variables (position variables).

b) Choose a sufficient number of revolute joints which when disconnected yield an open-loop system. Indicate those joints. Assign the joint variables and show them clearly.

c) Write the necessary number of independent loop closure equations using vectors described by directed lines such as $\overrightarrow{SQ}, \overrightarrow{RS}$, etc.

d) Using complex numbers, re-write these loop closure equations in terms of the joint variables and the fixed parameters of the mechanism.



$A_oA = a_2$, $AB = a_3$, $BC = a_5$

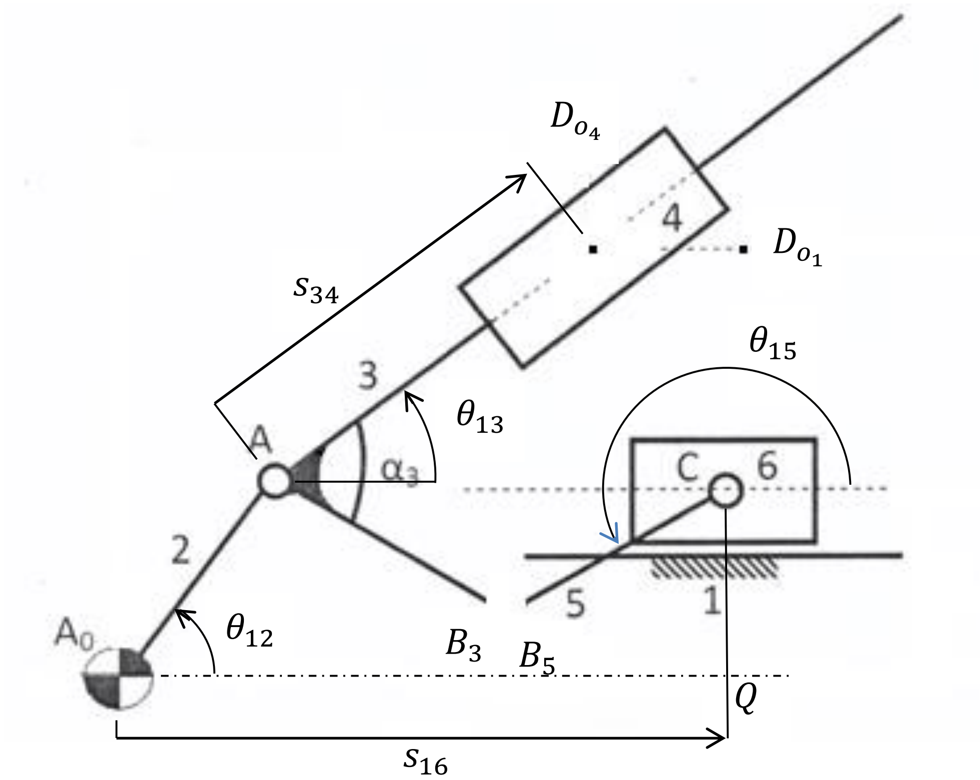
NOTE: The symbol ◕ indicates revolute joint of a link with the fixed link.

Solution:

a) $\ell = 6, j = 7\ (5R, 2P)\ \Rightarrow\ F = 3(6 - 7 - 1) + 7 = 1$

$L = 7 - 6 + 1 = 2$, # of joint variables$= 4 + 1 = 5$

b) R joints at B and D_o are chosen for disconnecting.



Joint variables for the remaining joints are θ_{12}, θ_{13}, s_{34}, s_{16} and θ_{15}.

c) LCE 1: $\overrightarrow{P_{B_3}} = \overrightarrow{P_{B_5}} \;\Rightarrow\; \overrightarrow{A_oA} + \overrightarrow{AB_3} = \overrightarrow{A_oQ} + \overrightarrow{QC} + \overrightarrow{CB_5}$

LCE 2: $\overrightarrow{P_{D_{o_4}}} = \overrightarrow{P_{D_{o_1}}} \;\Rightarrow\; \overrightarrow{A_oA} + \overrightarrow{AD_{o_4}} = \overrightarrow{A_oD_{o_1}}$

d) $\mathrm{a}_2 e^{i\theta_{12}} + \mathrm{a}_3 e^{i(\theta_{13}-\alpha_3)} = \mathrm{s}_{16} + ib_1 + \mathrm{a}_5 e^{i\theta_{15}}$

$\mathrm{a}_2 e^{i\theta_{12}} + s_{34} e^{i\theta_{13}} = \mathrm{a}_1 + i\mathrm{c}_1$

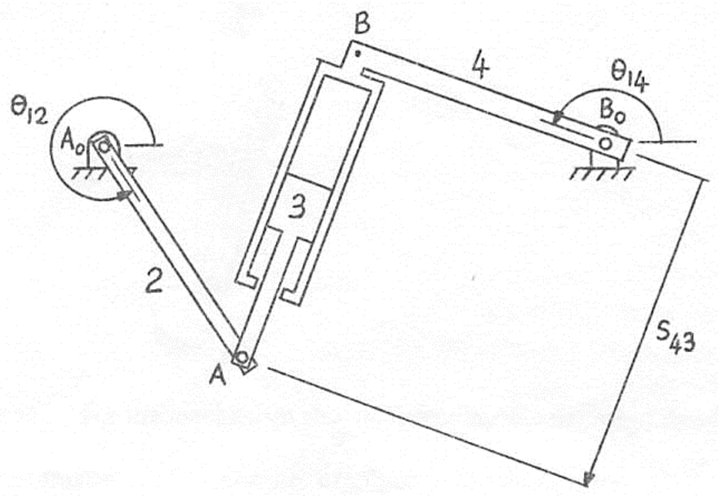
PROBLEM 2.

In the mechanism shown the link lengths are $A_oB_o = r_1 = 8\ cm$, $A_oA = r_2 = 4\ cm$ and $B_oB = r_4 = 4.5\ cm$ and the angle $B_oBA = 90^o$. s_{43} is the input variable. The loop closure equation is

$$\overrightarrow{A_oA} = \overrightarrow{A_oB_o} + \overrightarrow{B_oB} + \overrightarrow{BA}$$

Using analytical solution of loop closure equation find θ_{14} and θ_{12} when $s_{43} = 4\ cm$. Find <u>all</u> solutions corresponding to different assembly configurations of the mechanism.

Hint: First find θ_{14} by eliminating θ_{12}.



Solution:

$$r_2 e^{i\theta_{12}} = r_1 + r_4 e^{i\theta_{14}} + s_{43} i e^{i\theta_{14}}$$

Re: $r_2 \cos\theta_{12} = r_1 + r_4 \cos\theta_{14} - s_{43} \sin\theta_{14}$

Im: $r_2 \sin\theta_{12} = r_4 \sin\theta_{14} + s_{43} \cos\theta_{14}$

Square and add :

$$2r_1r_4 \cos\theta_{14} - 2r_1 s_{43} \sin\theta_{14} = r_2^2 - r_1^2 - r_4^2 - s_{43}^2$$

$$A\cos\theta_{14} + B\sin\theta_{14} = C$$

where $A = 2r_1r_4 = 72$, $B = -2r_1s_{43} = -64$, $C = r_2^2 - r_1^2 - r_4^2 - s_{43}^2 = -84.25$

$\Rightarrow D = \sqrt{A^2 + B^2} = 96.33$, $\varphi = atan_2(B, A) = -41.6^o$,

$$\theta_{14} = \varphi \pm cos^{-1}\frac{C}{D} = -41.6^o \pm 151.0^o = 109.4^o, -192.6^o$$

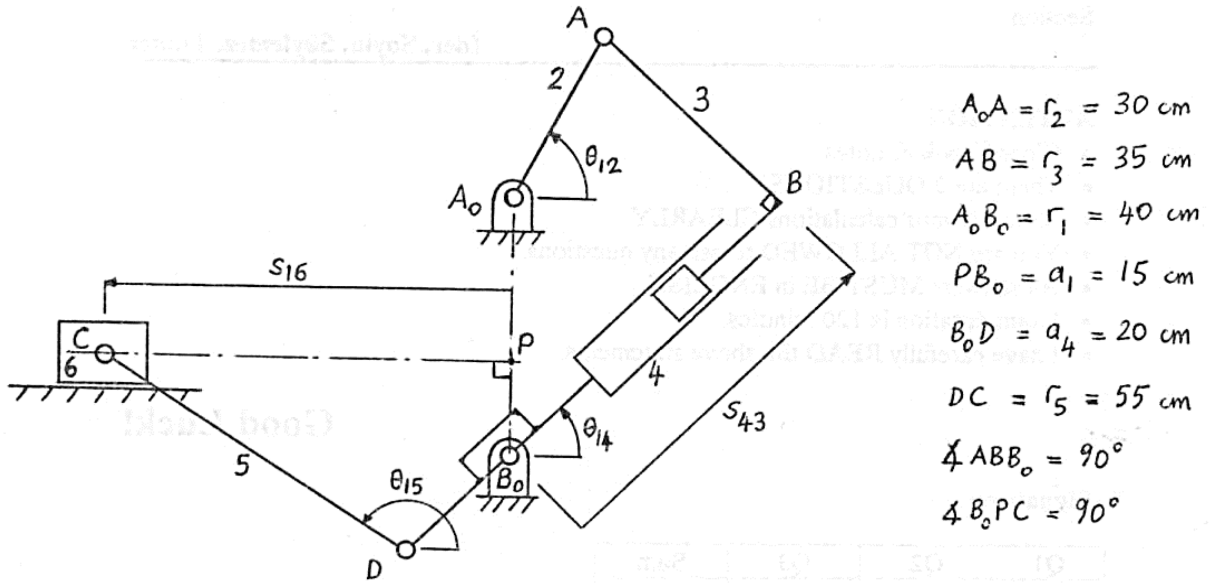
$$\theta_{12} = atan_2(r_4 \sin\theta_{14} + s_{43}\cos\theta_{14},\ r_1 + r_4\cos\theta_{14} - s_{43}\sin\theta_{14})$$

For $\theta_{14} = 109.4^o$, $\theta_{12} = atan_2(2.92, 2.73) = 46.9^o$

For $\theta_{14} = -192.6^o = 167.4^o$, $\theta_{12} = atan_2(-2.92, 2.73) = -46.9^o = 313.1^o$

PROBLEM 3.

The link lengths of a 6-link mechanism are given as shown. θ_{12} is the input variable. Using LCEs corresponding to loops $A_oABB_oA_o$ and B_oPCDB_o, find the dependent joint variables θ_{14}, s_{43}, θ_{15} and s_{16} when $\theta_{12} = 60^o$. Find the solutions of all assembly configurations.



Hint: First solve LCE corresponding to loop $A_oABB_oA_o$ for θ_{14} and s_{43}.

Solution:

LCE 1 : $\overrightarrow{A_0A} = \overrightarrow{A_0B_0} + \overrightarrow{B_0B} + \overrightarrow{BA}$

$$r_2 e^{i\theta_{12}} = -ir_1 + s_{43}e^{i\theta_{14}} + r_3 e^{i\left(\theta_{14}+\frac{\pi}{2}\right)}$$
$$= -ir_1 + s_{43}e^{i\theta_{14}} + ir_3 e^{i\theta_{14}}$$

LCE 2: $\overrightarrow{B_0P} + \overrightarrow{PC} = \overrightarrow{B_0D} + \overrightarrow{DC}$

$$ia_1 - s_{16} = a_4 e^{i(\theta_{14}+\pi)} + r_5 e^{i\theta_{15}}$$
$$= -a_4 e^{i\theta_{14}} + r_5 e^{i\theta_{15}}$$

Solution of LCE 1:

$$r_2 \cos\theta_{12} = s_{43}\cos\theta_{14} - r_3 \sin\theta_{14} \qquad (1)$$
$$r_2 \sin\theta_{12} = -r_1 + s_{43}\sin\theta_{14} + r_3\cos\theta_{14} \qquad (2)$$

To eliminate s_{43} rearrange eqs. (1) and (2) as follows:

$$s_{43}\cos\theta_{14} = r_2\cos\theta_{12} + r_3\sin\theta_{14} \qquad (3)$$
$$s_{43}\sin\theta_{14} = r_1 + r_2\sin\theta_{12} - r_3\cos\theta_{14} \qquad (4)$$

Taking the ratio of the two sides of eqs. (3) and (4):

$$\frac{\cancel{s_{43}}\cos\theta_{14}}{\cancel{s_{43}}\sin\theta_{14}}=\frac{r_2\cos\theta_{12}+r_3\sin\theta_{14}}{r_1+r_2\sin\theta_{12}-r_3\cos\theta_{14}}$$

Cross-multiplying gives

$$A\cos\theta_{14}+B\sin\theta_{14}=C \tag{5}$$

where

$$A=r_1+r_2\sin\theta_{12}$$
$$B=-r_2\cos\theta_{12}$$
$$C=r_3$$

If we let

$$A=D\cos\phi$$
$$B=D\sin\phi$$

where

$$D=\sqrt{A^2+B^2}$$
$$\phi=atan_2(B,A)$$

Eq. (5) reduces to

$$D\cos(\theta_{14}-\phi)=C$$

which yields

$$\theta_{14}=\phi\pm\cos^{-1}\left(\frac{C}{D}\right) \tag{6}$$

Eq. (6) gives two solutions for θ_{14}. These solutions correspond to different assembly configurations. Then for each θ_{14}, s_{43} can be found using either eq. (3) or eq. (4). Let us use Eq. (3):

$$s_{43}=\frac{r_2\cos\theta_{12}+r_3\sin\theta_{14}}{\cos\theta_{14}}$$

Solution of LCE 2 :

$$-s_{16}=-a_4\cos\theta_{14}+r_5\cos\theta_{15} \tag{1}$$
$$a_1=-a_4\sin\theta_{14}+r_5\sin\theta_{15} \tag{2}$$

To find θ_{15}, rearrange Eq. (2) as follows:

$$\sin\theta_{15}=\frac{a_1+a_4\sin\theta_{14}}{r_5} \tag{9}$$

or

$$\theta_{15}=\sin^{-1}\left(\frac{a_1+a_4\sin\theta_{14}}{r_5}\right) \tag{10}$$

There are two different solutions for the inverse sine function in Eq. (10). Again, these solutions correspond to different assembly configurations. For each θ_{15} found from Eq. (10), s_{16} can be found using Eq. (1) as follows:

$$s_{16}=a_4\cos\theta_{14}-r_5\cos\theta_{15} \tag{11}$$

The numerical results are tabulated below. Since each loop has two assembly configurations, the mechanism has four assembly configurations.

A= 65,98
B= -15
C= 35
D= 67,66

$$tan^{-1}\frac{B}{A} = -12.8^o, 167.2 \quad \Rightarrow \phi = -12.8^o$$

Configuration #	θ_{14} [°]	s_{43} [cm]	θ_{15} [°]	s_{16} [cm]
1	46,04	57,91	32,31	-32,60
2	46,04	57,91	147,69	60,37
3	-71,66	-57,91	-4,15	-48,56
4	-71,66	-57,91	184,15	61,15